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The Mayr-Pentegov Model of Electric Arc Involving the Use of the Exponential Function and Enabling the Approximation of Static Characteristics

Abstract: The article presents the justified use of functions containing the exponential component for the approximation of the static current-voltage characteristics of electric arc. The Author proposed a new function approximating the above-named characteristics which were next used in the mathematical Mayr-Pentegov model, expressed in two, i.e. differential and integral, forms. The two forms constituted the basis enabling the development of macromodels using controlled voltage and current sources. The article also presents families of dynamic current-voltage characteristics obtained through the simulation of processes in a circuit containing macromodels of arc powered by the source of current characterised by sinusoidal and trapezoidal waveforms and various frequency. The article demonstrates the usability of the proposed variants of the Mayr-Pentegov model.

Keywords: electric arc, static current-voltage characteristics, Mayr-Pentegov model.

Introduction

Complex mathematical models of arc may have wide ranges of applications in numerical calculations of electrotechnical (welding, electrothermal etc.) devices. However, the aforesaid models complicate the creation of macromodels, extend the time of numerical calculations and cause problems when determining parameters experimentally. Following the Pareto principle, taking into consideration all factors affecting the electric characteristics of arc has no significant justification as some of them are of little relevance. In turn, the simplified mathematical models of electric arc constitute a comfortable alternative useful for analytical deliberations and numerical calculations of power supply systems and control systems of electrotechnical equipment. However, because of numerous preliminary assumptions, the above-named models have relatively narrow areas of application. In the simplest mathematical models, the current-voltage characteristic is approximated using the power function [1, 2]. The characteristic is then monotonic, which in many cases, does not correspond to phenomena being observed. Depending on the presence of various factors, e.g. the shape and dimensions of electrodes, the pressure of plasma-forming

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gas etc., the static characteristics of arc may be non-monotonic [3].

The Pentegov-Sidorec model [4, 5] makes it possible to use a static characteristic of any shape to model electric arc. Such a characteristic can be relatively easy to determine by powering arc with direct current or periodic current having the symmetric rectangular shape. If arc is long (high-voltage), it is possible (in approximation) to dispense with the determination of near-electrode voltage drops. In turn, if arc is short (welding), the ignoring of the above-named voltage drops may lead to significant errors. Selected methods enabling the determination of the aforesaid drop are discussed in [6].

When creating the Pentegov-Sidorec model, adopted reduction assumptions lead to the linear equation of state. In the aforementioned equation, the value of the damping function (i.e. time constant) is constant. Publication [7, 8] describes a new improved Mayr-Pentegov model, in which dispensing with a certain reduction assumption resulted in the obtainment of a non-linear model, where the damping function depends on the conductance derivative in relation to squared current. Such an arc model enables the obtainment of simulation results closer to actual ones, yet it requires the use of the conductance derivative function. The facilitated use of the simulation possibilities of this model requires a set of functions approximating static current-voltage characteristics and corresponding characteristics of the conductance derivative.

Approximation of Static Current-Voltage Characteristics Using the Exponential Function

The static current-voltage characteristics of arc are approximated using various functions such as polynomials, power functions and exponential functions or their various combinations, e.g. ordinary sums of simple functions or sums using tapering functions [9–11]. Preferring the exponential function facilitates the determination of the coordinates of ignition voltage. However, an increase in the complexity of such functions impedes the creation of the Mayr-Pentegov model. As mentioned before, the reason is the necessity of the analytical determination of the conductance derivative. The article discusses the frequently used form of the approximation of the static current-voltage characteristics

$$U = U_0 \left(\frac{I}{I_0}\right)^m \exp\left(1 - \left(\frac{I}{I_0}\right)^m\right) + \rho I^n$$

where $m > 0$, $n \geq 0$, $U_0 + \rho I_0^n$ – approximate value of ignition voltage, I_0 – approximate value of ignition current. Because of relatively low value I_0, it is possible to adopt U_0 as ignition voltage. The inclusion of the component with the power function leads to the situation where if $n > 1$, a defined point (corresponding to ignition voltage) is shifted slightly towards higher voltage and lower current. In turn, if $n = 0$, ignition voltage is $U_0 + \rho$. Dependence (1) corresponds to the static conductance–current characteristic having the following form:

$$G(I) = \frac{I}{U} = \frac{I}{U_0 \cdot \left(\frac{I}{I_0}\right)^m \cdot \exp\left(1 - \left(\frac{I}{I_0}\right)^m\right) + \rho I^n}$$

It can be expressed as the following function:

$$G(I^2) = \frac{\sqrt{I^2}}{U_0 \cdot \left(\frac{\sqrt{I^2}}{I_0}\right)^m \cdot \exp\left(1 - \left(\frac{\sqrt{I^2}}{I_0}\right)^m\right) + \rho\left(\sqrt{I^2}\right)^n}$$

Its derivative is defined by the following dependence:

$$\frac{dG(I^2)}{dI^2} = \frac{\exp\left(\frac{I}{I_0}\right)^m \left[eU_0\left(m\left(\left(\frac{I}{I_0}\right)^m - 1\right) + 1\right)\left(\frac{I}{I_0}\right)^m - \rho(n-1)\exp\left(\frac{I}{I_0}\right)^m I^n\right]}{2I\left(eU_0\left(\frac{I}{I_0}\right)^m + \rho\exp\left(\frac{I}{I_0}\right)^m I^n\right)^2}$$

where $e = \exp(1)$.

Figure 1 presents the families of static current-voltage characteristics of electric arc corresponding to gradual changes in parameter m. In turn, Figure 2 presents the families of static characteristics corresponding to gradual changes in parameter n. It can be seen that proposed dependence (1) enables the obtainment of characteristics having complex shapes. In addition, it is possible to observe a certain similarity of these curves to the characteristics presented in publications [7, 8]. The diagrams of the conductance derivative are characterised by the rapid decrease in value near the zero value of current. In the remainder of the article it is shown that the values of state current close to zero are not obtained in the mathematical model. For this reason, it is necessary to use the values of the derivative with higher values of current, corresponding to the more gently decreasing fragments of the curves.

Mathematical Mayr-Pentegov model of the electric arc column

The Mayr-Pentegov model constitutes the system of two primary equations, one of which is differential or integral, whereas the other one is algebraic. The form of these equations determines the creation of various macromodels of arc using controlled voltage or current sources.

The variant of the model with the differential equation will be considered first. This is the non-linear first-order equation describing
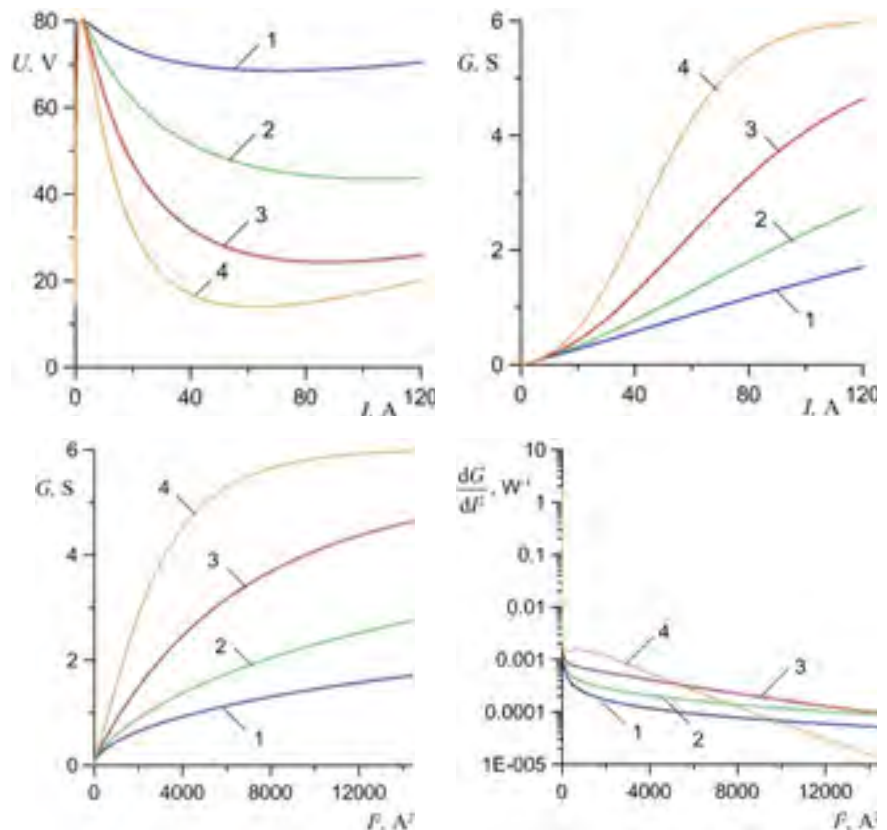

Fig. 1 Families of the static characteristics of electric arc: a) current-voltage characteristics; b) conductance–current characteristics; c) conductance–squared current characteristics and d) conductance derivative–squared current characteristics ($U_0 = 80$ V, $I_0 = 2$ A, $\rho = 0.1$ VA^{-n}, $n = 1.1$, 1-$m = 0.2$, 2-$m = 0.3$, 3-$m = 0.4$, 4-$m = 0.5$)
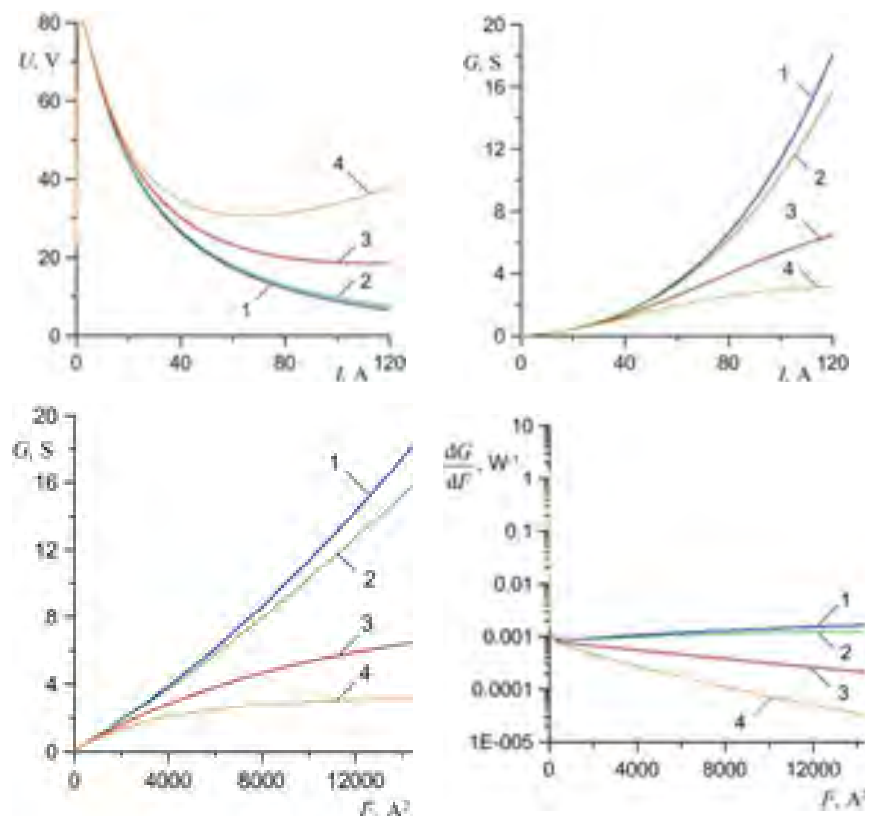

Fig. 2. Families of the static characteristics of electric arc: a) current-voltage characteristics; b) conductance–current characteristics; c) conductance–squared current characteristics and d) conductance derivative–squared current characteristics ($U_0 = 80$ V, $I_0 = 2$ A, $\rho = 0.1$ VA^{-n}, $m = 0.4$, 1-$n = 0.2$, 2-$n = 0.5$, 3-$n = 2$, 4-$n = 1.2$)

the dynamics of state current $i_\theta(t)$, corresponding to changes in plasma temperature

$$\theta(i_\theta, p)\frac{di_\theta^2}{dt} + i_\theta^2 = i^2$$

where the damping function is expressed as follows

$$\theta(i_\theta, p) = Q_p \frac{dg}{di_\theta^2}$$

The above presented function depends not only on state current i_θ, but also on the vector of parameters p. The designations introduced in the above-named equations include: i – excitation current, g – arc column conductance, Q_p – in the case of approximation, the exponential function is the subtangent of the diagram of function $g(Q)$, Q – plasma enthalpy. Equation (5) can be expressed in the following form:

$$i_\theta^2 = \int_0^t \frac{1}{\theta(i_\theta, p)}(i^2 - i_\theta^2)d\tau + i_{\theta 0}^2$$

The algebraic equation describes momentary changes of current u on the column

$$u = \frac{U(i_\theta)}{i_\theta}i$$

where $U(I)$ – static current-voltage characteristic. The mathematical model expressed using equation (7) and (8) corresponds to the modelling the non-linear resistance of the arc plasma column by means of the controlled voltage source having the arrow directed opposite to the flow of current. The second variant of the model uses the equivalent integral form of the Mayr-Pentegov model. Then, equation (5) can be expressed as presented below

$$i_\theta = i_{\theta 0} \exp\left(\int_0^t \frac{1}{2\theta(i_\theta, p)}\left(\frac{i^2}{i_\theta^2}-1\right)d\tau\right) =$$

$$= i_{\theta 0} \exp\left(\int_0^t \frac{1}{2\theta(i_\theta, p)}\left(\frac{u^2 g^2}{i_\theta^2}-1\right)d\tau\right)$$

Non-linear resistance is modelled by the controlled source of current having the value of source current

$$i = G(i_\theta^2, p)\cdot u = \frac{u i_\theta}{U(i_\theta, p)}$$

Its arrow is directed opposite in relation to voltage.

Resultant arc voltage $u_a(i)$ is the sum of voltage drops on plasma column u and in very thin near-electrode areas. The sum near-cathode voltage drop U_C and of near-anode voltage drop U_A is designated as U_{AC}. It is present regardless of power supply (with DC or AC). In the second case, depending on the properties of the electrode, the sum of voltage drops can be symmetric or asymmetric. In the symmetric case

$$u_a(i) = u(i) + U_{AC}\,\mathrm{sgn}(i)$$

whereas in the asymmetric case

$$u_a(i) = u(i) + \begin{cases} -U_{AC1}, & \text{if } i < 0 \\ U_{AC2}, & \text{if } i \geq 0 \end{cases}$$

Electrotechnical assumption of the macromodels of electric arc using controlled sources

During the macromodelling of low-voltage arc it is necessary to take into consideration the non-linearity of the characteristics of the arc column area as well as the non-linearity of the characteristics of the areas of near-electrode voltage drops. Because of the series connection of all three electric elements it is convenient to map them by means of the series connection of corresponding macromodels. Usually, two models of the near-cathode area and that of the near-anode area are modelled using one common model. The current-voltage characteristics of plasma in the near-electrode areas can be approximated using scaled functions similar to sign(i). For this reason, the simplest solution involves their modelling using ideal uncontrolled voltage sources. In accordance with the fundamentals of electrical engineering, the ideal voltage sources cannot be replaced with the ideal current sources and vice versa. Such an operation is only possible in cases of actual sources.

The transformation of a branch with the series connection of electric elements into parallel connected branches with equivalent dual elements is not always easy and possible. In cases of linear elements it is necessary to appropriately scale their values. If elements are non-linear, the principle of superposition does not apply and the development of dual elements is significantly impeded. Few publications on the subject [12] do not provide information on simple procedures.

The selection of the mathematical model (differential or integral) of the electric arc column affects the creation of a resultant computer macromodel:

– If the differential mathematical model of the electric arc column is used, it corresponds to the macromodel with the controlled voltage source. In such a case, it is not possible to connect in parallel the voltage source with other voltage sources. It is only possible to connect in parallel the voltage source with passive elements or current sources. In turn, it is possible to connect in series the controlled voltage source with passive elements and voltage or current sources.

– If the integral mathematical model of the arc column is used, it corresponds to the macromodel with the controlled current source. In such a case, it is not possible to connect in series the current source with other current sources. It is only possible to connect in series the current source with passive elements or voltage sources. In turn, it is possible to connect in parallel the controlled current source with passive elements and voltage or current sources.

Electric arc is powered by actual sources of electric energy. Such sources can be presented as actual current sources or their equivalent actual voltage sources (which is the case when testing both actual objects and using computer-aided simulations). Because of the non-linearity of electric characteristics, arc burning stability requires the powering of the circuit by current

sources having characteristics similar to ideal ones (with significant internal impedance). In spite of this, the obtainment of process stability during arc simulations characterised by various parameters and macromodel structures proves difficult. Physical arc terminates easily, which should correspond to the interrupted operation of a simulation programme.

The Author created a library containing macromodels of the column of arc expressed by various differential equations and their equivalent integral equations. Similarly, the Author created macromodels of near-electrode areas. The macromodel with the controlled voltage source was not problematic, yet the macromodel with the controlled current source required the adoption of additional assumptions. The model corresponded to the actual voltage source and could be used within the limited range of excitation. An opinion concerning the limited use of current sources for the macromodelling of arc and its diagnostics is presented in publication [13].

If it is assumed that near-electrode voltage drops correspond to actual voltage sources, their characteristics can be symmetric or asymmetric. Upon such an assumption, they can be transformed into actual current sources having symmetric or asymmetric characteristics. In the symmetric case

$$i(u) = G_{AC}(u - U_{AC} \operatorname{sgn}(u))$$

whereas in the asymmetric case

$$i(u) = \begin{cases} G_{AC1}(u + U_{AC1}), & \text{if } u < 0 \\ G_{AC2}(u - U_{AC2}), & \text{if } u \geq 0 \end{cases}$$

where G_{AC1} and G_{AC2} – tangents of the angles of the inclination of characteristics $i(\underline{u})$ of the current source, $U_{AC2} > 0$ V, $U_{AC2} > 0$ V. The further improvement of the stability of the simulation programme required the parallel connection of the current source with resistor R_{ac}.

Figure 3 presents four variants of designs of circuits containing connected macromodels of the source of current I_e powering electric

arc. The first variant with two controlled voltage sources is the most popular. It can be easily replaced with the variant containing one controlled voltage source. Significantly less popular is the variant of arc with the controlled current source with the non-linear characteristics of the arc column. The above-named variant is considered to be more favourable when simulating arc characterised by reduced stability, e.g. in gliding arc plasma torches [14]. The possibilities of using the third variant, with the controlled current source with non-linear plasma characteristics in near-electrode areas, were subjected to simulation.

Simulation of dynamic processes in a circuit with the Mayr-Pentegov model using the exponential function for the approximation of dependence U(I)

The investigation of the effectiveness of the modelling of electric arc using the above-presented mathematical dependences required
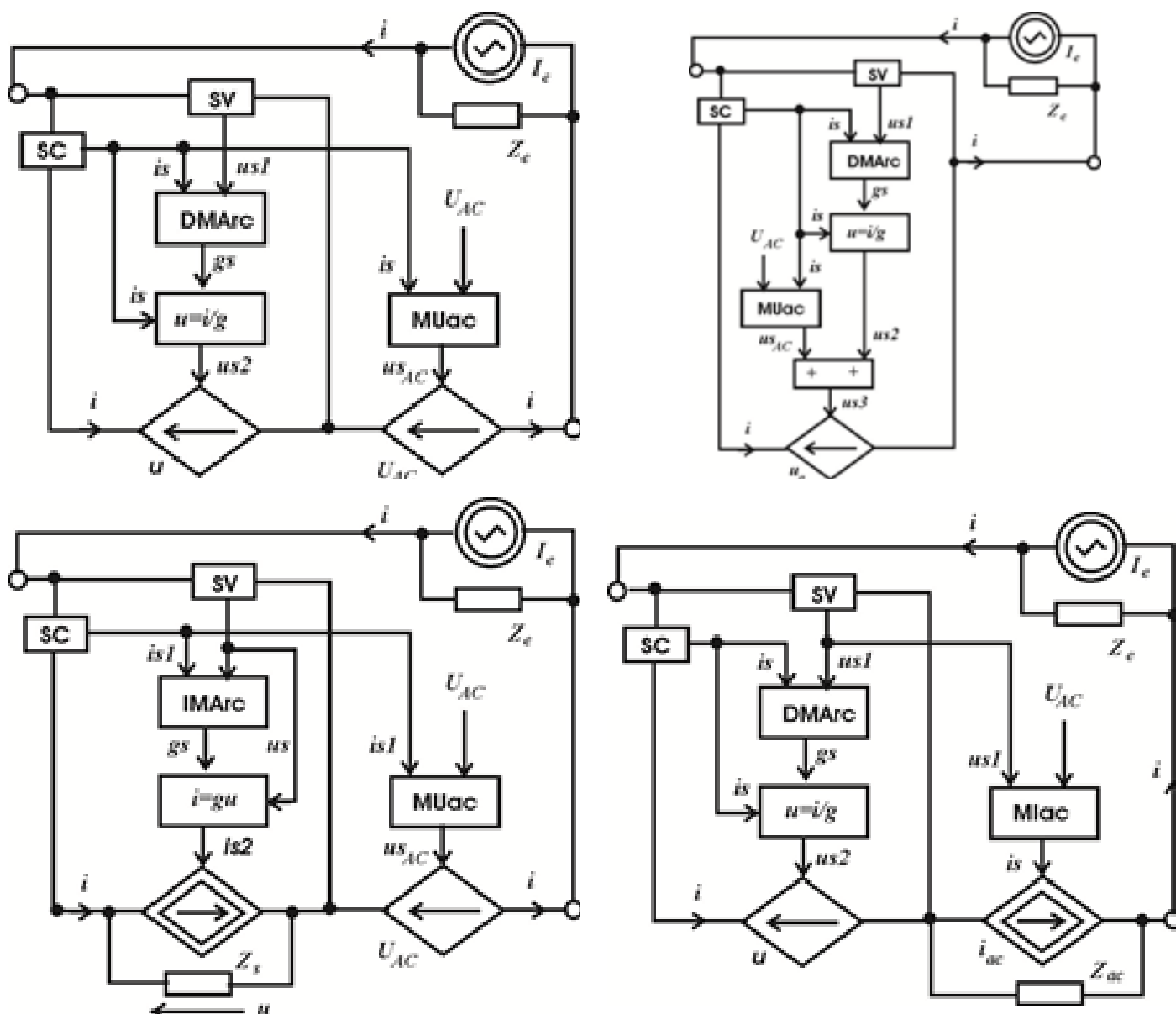

Fig. 3. Schematic diagrams of electric circuits composed of the power source and macromodels connected in series and modelling the quasi-homogenous areas of electric arc (SC – current sensor, SV – voltage sensor, DMArc – unit implementing equations (7) and (8), IMArc – unit implementing equations (9) and (10), MUac – unit implementing equation (11) or (12), MIac – unit implementing equation (13) or (14)): a) using the macromodels containing two controlled voltage sources; b) equivalent circuit (a) using a single voltage source; c) using the column macromodel containing the controlled current source and the macromodel of near-electrode areas containing the controlled voltage source; d) using the column macromodel containing the controlled voltage source and the macromodel of near-electrode areas containing the controlled current source.

the performance of simulations in a simple circuit featuring the current source and the macromodel of arc expressed by formulas (7), (8) and (11). In the first case, the amplitude of sinusoidal current amounted to 140 A, whereas its amplitude amounted to 25 Hz or 50 Hz. The assumption involved symmetric arc (11) and voltage drop U_{AC} = 18 V. The results of numerical calculations are presented in Figure 4. Afterwards, the tests were repeated using current having the trapezoidal waveform having an amplitude of 140 A and a frequency of 25 Hz or 50 Hz. In terms of the lower frequency, the slope of the rising and falling edge was $28 \cdot 10^4$ A/s, whereas in terms of the higher frequency, the slope of the rising and falling edge was $7 \cdot 10^5$ A/s. The assumption also involved asymmetric arc (12) with voltage drops U_{AC1} = 16 V and U_{AC2} = 8 V. The results of numerical calculations are presented in Figure 5.

Figure 6 presents the results of simulation tests in the circuit with arc macromodels containing mixed sub-macromodels of two different, i.e. voltage and current, types of controlled sources. The
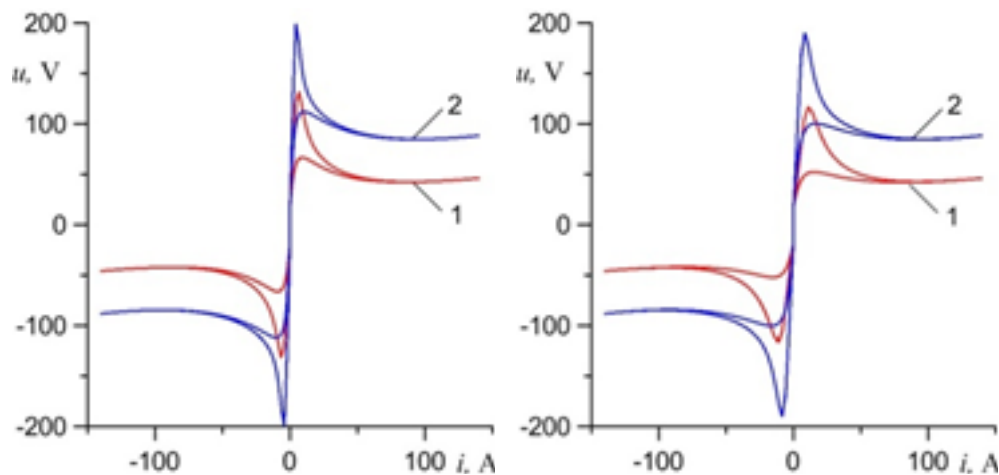

Fig. 4. Families of the dynamic current-voltage characteristics of arc powered by sinusoidal current: a) having frequency f = 25 Hz, 1- (U_0 = 80 V, I_0 = 2 A, ρ = 0.1 VA^{-n}, m = 0.4, n = 1.1, Q_p = 0.1 J), 2 - (U_0 = 120 V, I_0 = 2 A, ρ = 0.1 VA^{-n}, m = 0.3, n = 1.2, Q_p = 0.1 J) and b) having frequency f = 50 Hz, 1 - (U_0 = 80 V, I_0 = 2 A, ρ = 0.1 VA^{-n}, m = 0.4, n = 1.1, Q_p = 0.15 J), 2 - (U_0 = 120 V, I_0 = 2 A, ρ = 0.1 VA^{-n}, m = 0.3, n = 1.2, Q_p = 0.15 J)
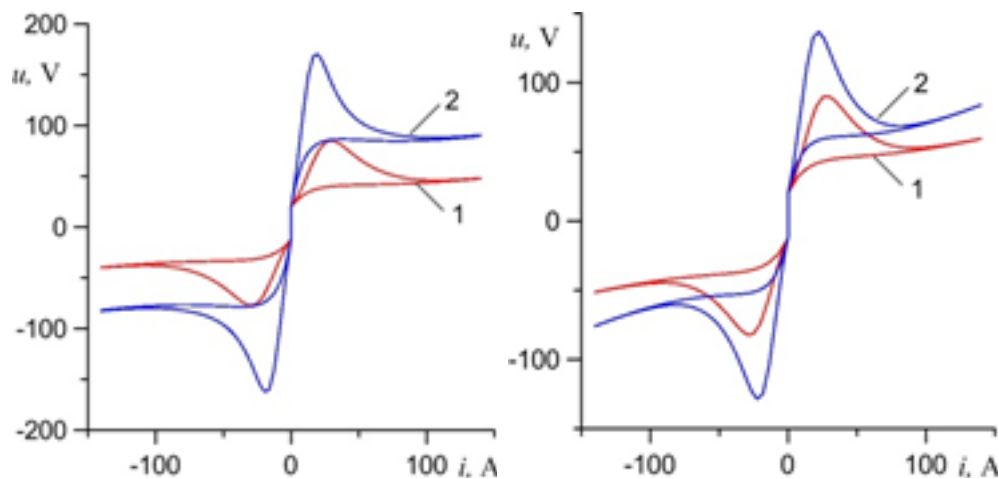

Fig. 5. Families of the dynamic current-voltage characteristics of arc powered by trapezoidal current: a) having frequency f = 25 Hz, 1- (U_0 = 80 V, I_0 = 2 A, ρ = 0.1 VA^{-n}, m = 0.4, n = 1.1, Q_p = 0.1 J), 2 - (U_0 = 120 V, I_0 = 2 A, ρ = 0.1 VA^{-n}, m = 0.3, n = 1.2, Q_p = 0.1 J) and b) having frequency f = 50 Hz, 1 - (U_0 = 80 V, I_0 = 2 A, ρ = 0.15 VA^{-n}, m = 0.4, n = 1.1, Q_p = 0.05 J), 2 - (U_0 = 120 V, I_0 = 2 A, ρ = 0.15 VA^{-n}, m = 0.4, n = 1.2, Q_p = 0.05 J)

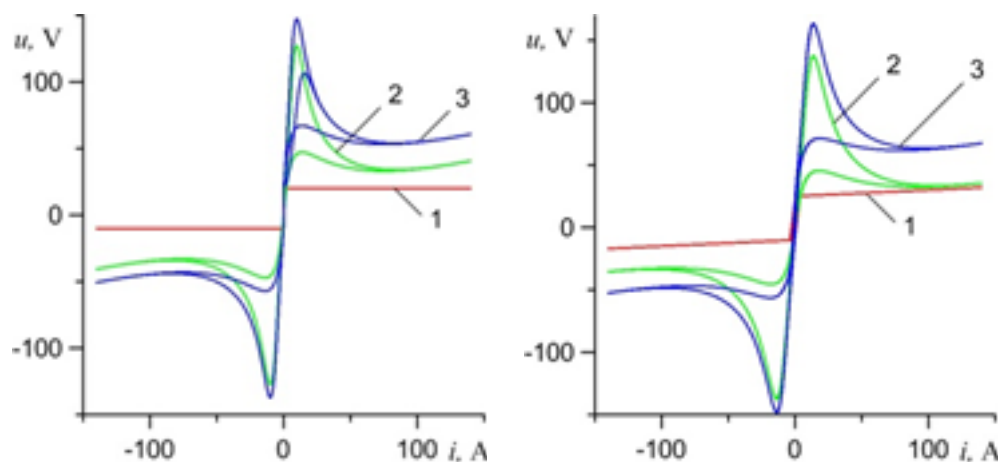

Fig. 6. Dynamic current-voltage characteristics of electric arc powered by sinusoidal current having frequency f = 50 Hz (U_{AC1} = 10, U_{AC2} = 25 V, 1 - U_{AC}, 2 – u_{col}, 3 – $U_{AC} + u_{col}$) a) parameters of the model (U_0 = 100 V, I_0 = 2 A, ρ = 0.15 VA^{-n}, m = 0.4, n = 1.1, Q_p = 0.3 J); b) parameters of the model (U_0 = 120 V, I_0 = 2 A, ρ = 0.2 VA^{-n}, m = 0.4, n = 1, Q_p = 0.3 J, $G_{AC1} = G_{AC2}$ = 20 S, $Z_{ac} = R_{ac}$ = 100 Ω)

results of the operation of such a circuit (Fig. 3c) using the integral model of the column (9) and (10) with the controlled current source and of the model of near-electrode areas with the controlled voltage source (11) are presented in Figure 6a. The use of the differential model of the column (5) with the controlled voltage source and of the model of near-electrode areas with the controlled current source (13) is significantly less favourable. The result of the simulation of processes in the circuit as is Figure 3d is presented in Figure 6b. To obtain the stability of simulation, the current-voltage characteristic of the near-electrode areas was deformed with the segments of the flat to the segments of the up-slope.

Based on the diagrams presented in Figure 6b it can be stated that the variant involving the use of the current source for the modelling of near-electrode voltage drops is less favourable than the previous ones as it increases the values of voltage on the macromodel of arc within the high-current range. The dynamic current-voltage characteristics presented in Figures 4, 5 and 6, indicate the extensive possibilities of modelling the electric properties of arc using the mathematical Mayr-Pentegov model with a selected function approximating static characteristics (1).

Concluding remarks

1. The advantage of the Pentegov-Sidorec and Mayr-Pentegov mathematical models is the possibility of (theoretically) selecting any static current-voltage characteristic. However, the second model uses the properties of the aforesaid characteristic to a greater extent and, as a result, enables the better mapping of electric arc.

2. Based on the dynamic characteristics it is possible to state that the use of the proposed exponential function-based (1) approximation in the Mayr-Pentegov model makes it possible to easily map the properties of electric arc burning under various physical conditions and within wide ranges of changes in current.

3. The extended set of functions approximating static current-voltage characteristics along with corresponding functions of conductance derivatives in relation to squared current may significantly facilitate the popularisation of the application of the Mayr-Pentegov model.

4. There are various possibilities enabling the creation of macromodels of arc using controlled voltage and current sources. Their selection depends on the required stability and accuracy of numerical calculations.

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